Can We Explain Thermodynamics By Quantum Decoherence?

Meir Hemmo* and Orly Shenker†

Can we explain the laws of thermodynamics, in particular the irreversible increase of entropy, from the underlying quantum mechanical dynamics? Attempts based on classical dynamics have all failed. Albert (1994a,b; 2000) proposed a way to recover thermodynamics on a purely dynamical basis, using the quantum theory of the collapse of the wavefunction of Ghirardi, Rimini and Weber (1986). In this paper we propose an alternative way to explain thermodynamics within no-collapse interpretations of quantum mechanics. Our approach relies on the standard quantum mechanical models of environmental decoherence of open systems, e.g. Joos and Zeh (1985) and Zurek and Paz (1994). © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Our experience tells us that macroscopic thermodynamic systems invariably evolve towards high-entropy states in an irreversible way. A central problem in the foundations of statistical mechanics (both classical and quantum) is to explain this experience by appealing to the underlying dynamics only. The aim here is twofold. First, to explain the macroscopic thermodynamical phenomena on the basis of the dynamical equations of motion that operate at the microscopic level, possibly using some probabilistic hypotheses. Then, to justify those probabilistic hypotheses by the same underlying dynamics. Hitherto neither of the two aims has been satisfactorily accomplished. Albert

*Department of Philosophy, University of Haifa, Haifa 31905, Israel (e-mail: meir@research.haifa.ac.il).
†Department of Philosophy, Logic and Scientific Method, London School of Economics, London WC2A 2AE, U.K. (e-mail: o.shenker@lse.ac.uk).

1See overviews of this problem in Sklar (1993) and Guttmann (1999).
(1994a,b, 2000) has proposed to solve this problem by appealing to the theory of the collapse of the quantum state by Ghirardi, Rimini and Weber (GRW) (1986). Albert argues that the genuinely chancy collapses postulated by the GRW theory entail the empirically observed behaviour of thermodynamic systems. If this were the only way to explain thermodynamics on the basis of quantum dynamics, the GRW approach would gain a serious advantage over its alternatives. This paper shows that this is not the case.

The aim of this paper is to propose an alternative way of explaining the laws of thermodynamics using the quantum mechanical dynamics in no-collapse theories. In particular, we shall consider the question of entropy increase on which Albert focuses. The main results we shall rely on are the standard models of so-called environmental decoherence of open systems (see Zurek (1982, 1993), Caldeira and Leggett (1983), Joos and Zeh (1985), Giulini et al. (1996) and references therein), and more recent results by Zurek, Paz and Habib (1993), and Zurek and Paz (1994, 1995)\(^2\) about the evolution of the von Neumann entropy in quantum mechanical (open) systems subjected to decoherence interactions. This paper, however, does not defend any specific no-collapse interpretation of quantum mechanics (e.g. pilot-wave, many worlds or modal theories).\(^3\) The results in decoherence theory (ibid.) strongly suggest that interactions with the environment are crucial in the emergence of quasi-classical and thermodynamic behaviour. The point we want to convey (and we do this by relying on a conjecture) is that in no-collapse interpretations of quantum mechanics these results might be taken to explain the approach to equilibrium and the second law of thermodynamics.

The paper is structured as follows. In Section 2 we present Albert’s approach to the problem using the GRW theory of the collapse of the quantum state. In Section 3, we give a brief description of the standard model of environmental decoherence, and then we describe Zurek and Paz’s results about the von Neumann entropy. Finally, in Section 4, we present our approach to the problem in the framework of no-collapse interpretations of quantum mechanics.

### 2. Albert’s GRW-based Approach

Albert proposes to explain thermodynamic phenomena by relying solely on the dynamics of the quantum state as prescribed by the GRW theory. Albert’s approach is manifestly Boltzmannian in that entropy is taken to be a property of the microstates (through their relation to macrostates) of individual systems. On his approach every single one of the possible initial (micro-)states of a thermodynamic system has a high probability to evolve thermodynamically,

\(^2\)See also Paz and Zurek (1999, Ch. 6, pp. 55–65).

\(^3\)See DeWitt and Graham (1973) on many worlds; Healey and Hellman (1998) and Dieks and Vermaas (1998) on modal theories; Cushing, Goldstein and Fine (1996) and Bub (1997) on the pilot-wave theory.
and therefore there is no need to postulate a specific initial state.\textsuperscript{4} The dynamics seems to be enough. As we will see Albert’s approach relies heavily on the fact that the GRW dynamics of the evolution of the quantum states is indeterministic.

Our empirical experience in thermodynamics suggests that irreversibility and the approach to equilibrium are law-like.\textsuperscript{5} That is, thermodynamic systems evolve to equilibrium universally and irrespective of their initial conditions. This general idea is applicable only in the right circumstances, and these ought to be taken into account. First, in systems having few degrees of freedom, fluctuations (which disagree with the predictions of thermodynamics) are dominant. The law-likeness of thermodynamics emerges only when we move to macroscopic systems with many degrees of freedom. Second, where quantum mechanical phenomena like superpositions dominate, thermodynamic magnitudes and their evolution are not always well-defined. For this reason, the explanation of thermodynamic behaviour crucially depends on the way the measurement problem is solved in quantum mechanics.\textsuperscript{6}

Hence, the explanation of thermodynamic phenomena in quantum mechanics is possible only in cases where the measurement problem is solved, and for macroscopic systems. These are the circumstances on which Albert focuses (as does our proposal, in Section 4, below).

We now describe Albert’s strategy. As the basic dynamical theory (in which the explanation of thermodynamics will be carried out) Albert proposes the GRW theory of the collapse of the wavefunction (see GRW (1986) and Bell’s (1987) discrete version). For our purpose it is enough to think about the GRW theory roughly as follows. Take the quantum mechanical wavefunction $|\psi(x_1 \ldots x_N, t)\rangle$ (written in the position basis) of some $N$-particle system. At almost all times it evolves in accordance with Schrödinger’s equation. But from time to time it gets multiplied by a narrow peaked Gaussian (approximately $10^{-5}$ cm wide) in position. This is known as the GRW collapse or jump of the wavefunction. The result of this multiplication is a new wavefunction for the system localised around the centre of the Gaussian. After such a collapse the new (normalised) wavefunction resumes its Schrödinger evolution, until the next jump occurs. These jumps are pure chancy events: the probability for a jump (for one particle in one second) is approximately $10^{-15}$. The probability that the jump localises the wavefunction around any given spatial point $x$ at time $t$ is given by the standard Born rule (i.e. by the squared inner product of the Gaussian and the wavefunction just prior to $t$). The two numbers above are taken by GRW to be new constants of nature. Given the choice of the

\textsuperscript{4}Special initial states seem to be required if the underlying dynamics is classical, as argued by Sklar (1973, p. 210) and repeated by Earman and Redei (1996).

\textsuperscript{5}Here we adopt the basic idea that the second law states entropy increase. In the literature even such basic questions as what is the approach to equilibrium, and what is irreversibility, are controversial; see Uffink (2001). It is also an open question whether the second law entails or assumes a time-asymmetric spontaneous evolution to equilibrium; see Brown and Uffink (2001, this issue).

\textsuperscript{6}In fact, we take it that without solving the measurement problem quantum mechanics has no empirical content at all, thermodynamic or otherwise.
constants above, the violation of conservation laws (e.g. energy, momentum) for macroscopic systems is practically unobservable. The measurement problem is solved by a straightforward application of the above dynamics to the quantum theory of measurement: superpositions of macroscopic pointer states are very quickly reduced with very high probability.

It is convenient to think about the GRW dynamics of the system as inducing random perturbations of its Schrödinger trajectory. In a sense, the GRW jumps may be taken to alter in a random way the Schrödinger trajectory the system would follow had the jumps not occurred. This means that the GRW trajectory can be thought of as a patchwork of segments of different Schrödinger trajectories each of which corresponds to a different initial state of the system. The net result of the GRW dynamics is that the trajectory of the system is genuinely and irreducibly stochastic with the probabilities given by the GRW constants above, and the usual quantum mechanical Born rule.

According to the GRW prescription, for any system and for any period of time there is a non-zero chance that a collapse will not occur at all. This chance gets smaller as the number of particles gets larger; but a case of no collapse is always possible. For future reference, call the (very probable) case where collapses onto approximate position eigenstates occur with relative frequency very close to the theoretical probabilities (i.e. the GRW chances for a collapse), GRW-normal evolutions; and the (improbable) cases where collapses do not occur in this way, GRW-abnormal. Albert focuses on GRW-normal evolutions only. In the GRW-abnormal evolutions thermodynamic magnitudes (and their statistical mechanical counterparts, e.g. kinetic theory of dilute gases) will not, in general, be well-defined, but these evolutions are extremely improbable.

Among the GRW-normal evolutions, Albert further distinguishes between thermodynamic-normal and abnormal evolutions. This is simply the distinction between evolutions that obey the laws of thermodynamics and those that do not. We call them normal and abnormal because statistical mechanics predicts (or, rather, aims to demonstrate) that the thermodynamic evolutions are very probable and the anti-thermodynamic evolutions are very improbable. We note that Albert works explicitly with a Boltzmannian notion of entropy according to which entropy is a property of microstates in virtue of the macrostates to which they belong, rather than of ensembles or probability distributions, as in the Gibbsian approach. Boltzmannian entropy can change in the closed systems on which Albert focuses.

Let us now draw the connection according to Albert’s approach between GRW-(ab)normality and thermodynamic-(ab)normality. We want to determine whether the time evolution of a given system is thermodynamic-normal or abnormal. Consider, for instance, a gas spreading out in the container following the removal of a partition. At \( t_0 \), when the partition is removed, the composite wavefunction of the gas molecules is

\[
|\Psi(0)\rangle = \sum_i \lambda_i(0)|\Phi_i\rangle,
\]

(1)
where the $|\Phi_i\rangle$ are some wavefunctions in position representation and the $\lambda_i(0)$ are the corresponding quantum mechanical amplitudes. Suppose, now, that the system follows a GRW-normal evolution. This means that its wavefunction evolves in time in accordance with the GRW dynamics, so that it actually undergoes collapses, with a rate that is close to the GRW chances. In particular, it collapses onto a state that is localised around a certain position $x = x_1, x_2, \ldots, x_N$, that is around some spatial distribution of the gas molecules. For example at $t_1$ it will have collapsed into some state $|\psi_1\rangle$ which corresponds to a Gaussian centred around $x(t_1)$. Then, the collapsed state evolves according to the Schrödinger equation until the next collapse, which occurs at $t_2$. In practice, the interval $(t_1, t_2)$ between collapses is extremely short for macroscopic systems like our gas, and $|\psi_1\rangle$ does not have much time for its Schrödinger evolution before the next collapse. Still, that short time is enough to make sure with almost certainty that the next collapse, at $t_2$, will not be centred around $x(t_1)$, but around a different position $x(t_2)$.\footnote{Recall that $x(t_2)$ depends probabilistically on the quantum wavefunction. Because of the tails of the GRW Gaussian there is a small but non-zero probability that the GRW jump at $t_2$ will hit a point far away from the previous point. This is a special case of what we have called GRW-abnormal evolutions.}

By our supposition, the system evolves in a GRW normal way, and the result is an effective trajectory for the system which in principle can be described in terms of thermodynamic magnitudes. It is therefore possible, in this case, to determine whether the evolution is thermodynamic-normal or abnormal (that is, whether it is thermodynamic or anti-thermodynamic). Had the system evolved in a GRW-abnormal way, its thermodynamic properties would not always be well-defined. For instance, in such cases the gas molecules may not have a determinate position. In this sense, GRW-normality is a pre-condition for determining whether or not a given evolution is a case of thermodynamic-normality. Therefore, henceforth we focus on GRW-normal evolutions only.

Suppose now that we write down the GRW equation for a given thermodynamic system, and solve it for all possible initial states. Consider any time interval $(t_1, t_2)$ for which the GRW evolution is normal (i.e. a GRW collapse occurs with high probability). For every possible initial state at $t_1$ there are many (possibly infinitely many) possible final states at $t_2$, which arise by a GRW-normal evolution. For each such evolution, it is then possible to determine whether the evolution is thermodynamic or anti-thermodynamic. Albert now proposes the following hypothesis. For each and every possible initial state, the thermodynamic evolutions starting from it vastly outnumber the anti-thermodynamic evolutions (among the GRW-normal ones).

Let us emphasise the status of this hypothesis. The thermodynamic-normality of the overwhelming majority of evolutions for any initial state has not yet been proved in the GRW theory. Albert proposes the hypothesis that solving the GRW equations of motion for thermodynamic systems will yield the above result. (Hence, perhaps, the use of vague terms like ‘vastly
outnumber’. In this sense it is an empirical hypothesis, but the actual proof of it is left for future work.\(^8\)

According to Albert the GRW jumps should be thought of as genuinely random perturbations of the quantum state. By the above hypothesis, the GRW perturbations are overwhelmingly likely to put a system on a segment of a Schrödinger evolution that is thermodynamic-normal, no matter whether or not the previous evolutions have been thermodynamic-normal or abnormal. This means that in the GRW theory the property of being thermodynamic-normal is stable over time, and that of being thermodynamic-abnormal is highly unstable. Thus the outcome is an effectively thermodynamic-normal evolution, for each and every possible initial state.\(^9\)

Since the dynamics is genuinely stochastic, to achieve this result there is no need to invoke postulates regarding the initial state of the system. Thus Albert provides a possible way of explaining the laws of thermodynamics on the basis of the underlying GRW dynamics only.\(^10\) Moreover, in Albert’s approach there is no need for two kinds of probabilities in physics: namely, in statistical mechanics and in quantum mechanics. We have only one origin of probability, which is quantum mechanical, and which can be understood as pure chance. We re-emphasise that Albert’s hypothesis, although supported both by the thermodynamic phenomena and by Boltzmannian reasoning, still awaits a proof, i.e. a proof that the preponderance of thermodynamic-normal evolutions is a consequence of the GRW dynamics.

### 3. Quantum Decoherence and Thermodynamics

We will now consider an alternative approach to Albert’s; namely, the recovery of thermodynamic behaviour in quantum mechanics without collapse, and the crucial role played by environmental decoherence in this approach.\(^11\) We start by very briefly describing the standard models of environment-induced decoherence in no-collapse quantum mechanics (see e.g. Zurek (1982, 1993), Caldeira and Leggett (1983), Joos and Zeh (1985), Giulini et al. (1996), Paz and Zurek (1999, Chs 2–5)).

\(^8\) Albert gives some plausibility arguments, but no detailed calculations have been carried out so far; see Albert (2000, pp. 148–162, and especially p. 155).

\(^9\) Hard cases for Albert are thermodynamic systems in which the frequency of the GRW jumps is too low (e.g. thin small gases), or in which the increase of entropy is actually reversed (e.g. the ‘spin-echo’ experiments). In these cases Albert appeals to, respectively, the GRW collapses in the environment of those systems (e.g. the containers of the gases), or in the system’s previous history. See Albert (2000, pp. 156–159).

\(^10\) But note that given the GRW theory one still needs to assume a low-entropy initial state in the past (Albert, 2000, pp. 161–162). This assumption has to be added by hand since it is not retrodicted by the GRW theory.

\(^11\) In this respect our approach belongs to the interventionist (or open systems) tradition in the foundations of classical statistical mechanics; see Ridderbos and Redhead (1998), Sklar (1993), Blatt (1959), Bergmann and Lebowitz (1955).
In the standard quantum mechanical models of decoherence the total initial state of a macroscopic system plus environment is usually assumed to be a product state

$$|\psi(x_1 \ldots x_N, t)\rangle \otimes |E\rangle,$$

(2)

where $|\psi(x_1 \ldots x_N, t)\rangle$ is the quantum state of the system and $|E\rangle$ is some state of the environment. In the interaction there is some position (or approximate position) observable $\Pi$ of the system that commutes (approximately) with the interaction Hamiltonian $H_{\text{int}}$:

$$[H_{\text{int}}, \Pi] \approx 0.$$

(3)

$\Pi$ is called the **decohering (pointer) variable** of the system. Taking an eigenbasis of this variable, the time-evolved (Schrödinger) state can be written in the form

$$|\psi(t)\rangle = \sum_i \mu_i(t) |\psi_i\rangle \otimes |E_i(t)\rangle,$$

(4)

where the kets $|\psi_i\rangle$ are the eigenstates of $\Pi$, and the $|E_i(t)\rangle$ are the relative states of the environment. The set of states $\{|\psi_i\rangle\}$ is called the **pointer basis**. The result of the coupling is that the scalar products between the environment states $|E_i(t)\rangle$ in (4) relative to different pointer states $|\psi_i\rangle$ decay exponentially, satisfying

$$\langle E_i(t + \Delta t) | E_j(t + \Delta t) \rangle \approx \delta_{ij}$$

(5)

after extremely short times $\Delta t$ (called **decoherence times**) which are typically around $10^{-23}$ sec. The decay of the scalar products in (5) is known as **environmental decoherence**.

From (5) and (4) it follows that the reduced state of the decohering system approaches the diagonal form:

$$\rho_s(t) \approx \sum_i |\psi_i\rangle \langle \psi_i| \langle \psi_i| \mu_i(t) \rangle^2 |\psi_i\rangle,$$

(6)

within times comparable to $\Delta t$. This form of (6) is stable as long as the scalar products (5) remain vanishingly small. Joos and Zeh (1985) derive a master equation for the reduced state of the system assuming recoil-free scattering (e.g. large mass ratio of the decohered system over the scattered particles) and isotropy in the distribution of the incoming particles (photons and molecules). Under these assumptions, the solutions of the equation exhibit exponential decay of the off-diagonal elements. The localisation rate is proportional to $e^{-A(x - y)^2}$, thus depending on various factors, such as the strength of the coupling, temperature and mass ratios. Zurek, Habib and Paz (1993) consider the decoherence interaction of a harmonic oscillator with an environment in thermal equilibrium. They show explicitly in the weak-coupling limit that the pointer states $|\psi_i\rangle$ correspond to so-called **coherent states**, i.e. narrowly peaked Gaussians in both position and momentum. In their model, coherent states are

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12 The relaxation times of the system are typically extremely long, in some models of the order $10^{40}$ sec.
the most stable states for the system in the sense that they produce the least (von Neumann and linear) entropy ($\rho_s(t)$ becomes maximally mixed when diagonalised by coherent states). In this sense one can say that decohering systems in these models follow quasi-classical trajectories.

In the context of this paper it is important to stress that the above models do not strictly apply to individual gas molecules, as in our example of the gas expanding in a box. In fact, decoherence of a gas molecule does not lead to localisation of the molecule’s wavefunction. In the case of a gas, collisions between the molecules and scattered particles do lead to decoherence, but they also distort (due to recoil) the trajectories of the molecules. In particular, the interactions result in delocalisation of the molecules’ wavefunction. This means that position in this kind of interaction does not correspond to a stable pointer basis for the molecules. And so one cannot use decoherence in order to recover quasi-classical trajectories of molecules. However, in typical cases (e.g. a diluted gas) decoherence does yield the same function of state (and the two-point correlations) as calculated in the classical kinetic theory. In this sense the classical predictions are recovered. We now propose the following conjecture: namely, that the decoherence interactions in the case of a gas induce perturbations of the molecules’ wavefunction that are enough to put the gas with high probability on thermodynamic-normal trajectories. We shall have to rely on this conjecture in our decoherence-based proposal for the recovery of thermodynamics (see Section 4).

Zurek and Paz (1994, 1995) (see also Paz and Zurek (1999, Ch. 6, pp. 55–65)) consider the connection between decoherence and classical chaos. They compare the evolution of an open quantum system with that of a closed system. For closed systems, as a result of the chaotic instability, the classical evolution of a phase space distribution (the Liouville flow) becomes highly striated on finer length scales. The Wigner function (i.e. the phase space distribution function obtained from the density matrix) cannot follow this evolution, since it cannot be positive throughout $\hbar$-sized regions of phase space. The Wigner function develops interference terms (the high derivative quantum correction terms) on finer length scales which very quickly become comparable to the Poisson bracket (which describes the classical Liouville flow of the system). This means that the resulting quantum evolution is in general significantly different from the classical evolution.

This situation changes when the system is subjected to a decoherence interaction with the environment. In this case there is an additional term in the time evolution of the Wigner function which produces diffusion. Zurek and Paz (1994, 1995) (see also Paz and Zurek (1999, pp. 58–60)) show that this term suppresses the quantum coherence of a wavepacket on longer length scales.

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13 We thank Professor Dieter Zeh for explaining to us this issue of decoherence of gas molecules.

14 Here, interference terms arise as a result of the dynamical instability due to chaos: roughly, the Wigner function gets exponentially stretched in position and contracted in momentum. See Zurek, Habib and Paz (1993), Zurek and Paz (1994, 1995) and Paz and Zurek (1999, pp. 56–58).
The interference terms in the reduced dynamics are rapidly washed out (in correspondence with (6)), and the different terms in the Wigner function become dynamically independent of each other. The spreading in position (due to diffusion) and the contraction in momentum (due to chaos) are cut off, roughly, when the rate of decoherence and the rate of divergence of trajectories are comparable. This happens when the Wigner function spreads over a critical coherence length \( \ell_c \),

\[
\ell_c = \frac{h\sqrt{\lambda}}{2d},
\]

where \( \lambda \) is the Lyapunov exponent (denoting, roughly, the time scale for divergence), and \( d \) is the diffusion coefficient in the Wigner function. At this point, the classical Poisson bracket provides an approximate description of the evolution of the Wigner function. In this sense the reduced dynamics of the quantum system recovers (approximately) the classical dynamics.

Let us consider now the connection between entropy increase and decoherence of open systems. Zurek, Habib and Paz (1993) have shown that the von Neumann entropy \( -k\text{Tr} \rho \ln \rho \) is a monotonic function of the volume of phase space over which the Wigner function spreads. Zurek and Paz (1994, 1995) show that for regular (non-chaotic) systems the increase of the von Neumann entropy depends linearly on the value of the diffusion coefficient throughout the time evolution. This is matched by the slow increase in phase space volume filled out by the Wigner function. For a chaotic system, however, the dependence on the diffusion coefficient is washed out after an initial transient: on longer time scales (but before equilibrium) the rate of entropy production is fixed solely by the (chaotic) dynamics. These results are reconfirmed by Monteoliva and Paz (2000). The number \( N \) of eigenstates of \( \rho \) increases as:

\[
N \simeq \frac{\ell(t)}{\ell_c} \approx \frac{h}{\Delta p_0 \ell_c},
\]

where \( \ell(t) \) is the coherence length, and \( \Delta p_0 \) is the initial spread in momentum. Assuming, for instance, that the diagonal elements of \( \rho_s(t) \) in (6) are approximately equal, the von Neumann entropy is \( S \simeq \ln N \). Given (8), the rate of increase of the von Neumann entropy is

\[
\dot{S} \approx \frac{d}{dt} \ln \left( \frac{\ell(t)}{\ell_c} \right) \approx \lambda,
\]

that is, roughly, the von Neumann entropy increases at a rate which is approximately equal to the divergence rate of trajectories of the chaotic system. Taking into account that decoherence destroys interference terms on finer length scales of the order of \( h/\ell^{(i)}(t) \), Zurek and Paz (1995; 1999, p. 62) show that the entropy increase in (9) asymptotically approaches

\[
\dot{S} \approx \sum_i \lambda^{(i)}_+,
\]

where the sum is over all the degrees of freedom of the system, and \( \lambda^{(i)}_+ \) denote the corresponding (small-scale) positive Lyapunov exponents. That is, the
Kolmogorov–Sinai rate for entropy production is recovered. Note that although the net result is that for longer time scales entropy production is independent of the value of the diffusion coefficient, the diffusion (and thereby decoherence) is essential in determining the coherence length $\ell_c$. We now discuss the implications of this result in the context of the foundations of statistical mechanics.

4. Discussion

The above results by Zurek are, indeed, remarkable. However, they do not yet provide a full explanation of the thermodynamic phenomenon of entropy increase. Several problems remain which we now consider. First, decoherence by itself does not solve the measurement problem in quantum mechanics. The interference terms in the superposition (4) are not eliminated, but rather diffused into the degrees of freedom of the environment. It is true that the effects of interference between the different terms in (4) are effectively undetectable for times longer than the decoherence time $\Delta t$ of the system. In the pointer basis the reduced state $\rho_s(t)$ does have the form of a classical statistical mixture. But $\rho_s(t)$ in (6) is an improper mixture. And this means that the diagonal elements $|\mu_i(t)|^2$ cannot be interpreted as probabilities of the corresponding states $|\psi_i\rangle$. In particular, decoherence is not enough to explain why our experience singles out, say in measurement situations, only one of the $|\psi_i\rangle$ as actually occurring on each occasion.

Second, the connection between the von Neumann entropy and the ideas of statistical mechanics is unclear: since $\rho$ is not a probability distribution, the von Neumann entropy cannot be given a Gibbsian interpretation in terms of a probability distribution. Similarly, one cannot meaningfully construct a Boltzmannian magnitude based on dividing the diagonal elements in $\rho$ into sets corresponding to macrostates. In a Boltzmannian approach entropy is a physical relation between a given microstate of an individual system and a given macrostate. In classical statistical mechanics a macrostate is associated with a volume in phase space, and the entropy of the microstate of the system at a given time is the logarithm of the standard measure of the volume which includes this microstate at this time. Thus, it is part and parcel of the Boltzmannian notion of entropy that the system actually be in a given microstate (and a fortiori in a given macrostate). In quantum mechanics without collapse this requires a solution to the measurement problem.

We shall now argue that the above problems may be solved by appealing to no-collapse interpretations of quantum mechanics. Such interpretations are modal, many worlds, and pilot-wave.

\begin{enumerate}
\item[A problem we do not address is that the von Neumann entropy does not straightforwardly correspond to the entropy of thermodynamics; see Shenker (1999).]
\item[In fact, in a theory without collapse $\rho_s(t)$ becomes more and more (improperly) mixed as a consequence of the decoherence interaction itself.]
\item[Such interpretations are modal, many worlds, and pilot-wave.]
\end{enumerate}
there are extra dynamical laws (over and above the Schrödinger equation) according to which \( \rho_s(t) \) in (6) represents a genuine probability distribution over the \( |\psi_i\rangle \), the state of the system at each time is associated with one of the states \( |\psi_i\rangle \) (call them effective states) corresponding to the diagonal elements of \( \rho_s(t) \), and there are transition probabilities between any two such effective states at different times.\(^{18}\) The role of decoherence in such interpretations is to explain (on the basis of the dynamics of the quantum state) why the different terms in the time-evolved superposition (4) effectively cease to interfere.\(^{19}\) And then the diagonal form of \( \rho_s(t) \) (as in (6)) together with the interpretation of \( \rho_s(t) \) as describing probabilities can explain in such interpretations the so-called effective collapse of the state. Consider now how the phenomenon of entropy increase (and the Zurek and Paz results) may be explained in such interpretations.

Consider first systems which conform to the standard models of decoherence (e.g. Caldeira and Leggett (1983), Joos and Zeh (1985), Zurek, Habib and Paz (1993)). In these models the total state of system plus environment has the form (4), that is

\[
|\Psi(t)\rangle = \sum_i \mu_i(t) |\psi_i\rangle \otimes |E_i(t)\rangle,
\]

where the effective states \( |\psi_i\rangle \) diagonalising the reduced state of the system are coherent states. Take some dynamical time interval \((t_1, t_2)\) longer than the decoherence time of the system. The total state above evolves in accordance with the Schrödinger equation. The coherent states in these models are the maximally stable states under the time evolution (including the decoherence interaction). The different terms in the superposition (11) which are approximately product states of the form

\[
|\psi_i\rangle \otimes |E_i(t_1)\rangle
\]

evolve approximately in accordance with the Schrödinger free evolution (essentially, because the \( |E_i(t_1)\rangle \) do not re-interfere). If the time evolution induces some spread in position (due to changes in constraints), the reduced state of the system will become mixed in each of the time-evolved terms (12), but decoherence insures that it will be diagonalised, again, by coherent states. And so in all branches of the total state the new effective states of the system at \( t_2 \) are coherent states coupled to (approximately) orthogonal \( |E_i(t_2)\rangle \). The states \( |\psi_i\rangle \) at \( t_2 \) are now centred around some spatial points \( x_i(t_2) \) that, in general, are different from \( x_i(t_1) \).

We may now assume that a single effective state at \( t_1 \) (i.e. one of the \( |\psi_i\rangle \) at \( t_1 \)) does not uniquely determine a single effective state at \( t_2 \). That is, we assume

\(^{18}\)The interpretation of the reduced state as describing the single-time probability distribution over the \( |\psi_i\rangle \) is similar in the pilot-wave, modal and many worlds theories. However, these theories differ in their account of the multi-time (joint and transition) probabilities.

\(^{19}\)Note that the pointer basis which is taken here as preferred in decomposing \( \rho_s(t) \) is fixed by the condition in (3).
that the transitions between the effective states are genuinely stochastic (the two-time correlations are not one-one). This depends on the details of the extra dynamics of the no-collapse theory in question: in some modal and many worlds interpretations the dynamics of e.g. a gas molecule for times shorter than the decoherence time of the molecule, is genuinely stochastic. The result is that in such a theory the effective state of the thermodynamic system changes in the course of decoherence stochastically, as if it undergoes random perturbations. These transitions do not in fact depend on initial conditions (over and above those needed to secure decoherence). In this sense they play a role similar to the GRW jumps in Albert’s approach.

Suppose now that we write down the Schrödinger equation for a given thermodynamic system, say our gas system, and solve it for all possible initial states. Take any two points of time $t_1$ and $t_2$ comparable with the decoherence time of the system. For every possible initial state at $t_1$ (which we assume is approximately pure) there are many possible evolutions that branch out from it, corresponding to different relative states of the environment (many possible final states at $t_2$). Compare all the pairs of states, one of which is a possible initial state at $t_1$ and the other is one decoherence evolution of it at $t_2$. It is then possible to determine, for each such pair, whether the transition from the effective state at $t_1$ to the state at $t_2$ is thermodynamic or anti-thermodynamic.

The above analysis suggests the following. First, the von Neumann entropy is a function of $\rho$, and so it belongs to a Gibbsian approach to statistical mechanics. Zurek and Paz’s results suggest that decoherence brings about an increase of Gibbsian entropy. Given the proximity between the results of applying the Gibbsian and Boltzmannian approaches in the right circumstances, it is highly reasonable that entropy in a Boltzmannian approach (in which Albert works) increases as well due to decoherence. That is, it is highly reasonable that the most probable macrostates (as determined by the diagonal of $\rho_s(t)$) correspond to high thermodynamic entropy. But recall that the above analysis applies only to cases in which a pointer basis is fixed by the decoherence interaction. This is not the case for gas molecules which are not localised by decoherence. For this case we need to appeal to our conjecture (Section 3) that the perturbations of the molecules in the decoherence interaction are enough to put them with high probability on thermodynamic-normal trajectories. If this conjecture can be proved, one could say that the

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20 See Bacciagaluppi (1998) for a detailed discussion of the modal interpretation of quantum mechanics, and in particular the extra stochastic dynamics. In the pilot-wave theory the velocity equation is deterministic, and so the trajectory of the system is fixed by the initial conditions and the dynamics.

21 Albert (2000, pp. 152–153) argues that the stochastic dynamics of the extra variables in modal interpretations will not in general induce the right transitions required for thermodynamic evolutions. This is correct as long as the thermodynamic system is isolated. But, as argued above, in no-collapse theories the effects of decoherence on the stochastic transitions are crucial for the explanation of thermodynamic evolutions.

22 But the results are not identical, as emphasised by Jaynes (1965).
stochastic transitions in no-collapse theories are enough to explain entropy increase and the approach to equilibrium on the basis of the dynamics alone regardless of initial conditions.\textsuperscript{23}


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References


\textsuperscript{23}With the exception of the pilot-wave theory in which initial conditions are needed. Also note that we need to assume a low-entropy initial state.


